Targets and Limits for Management of Fisheries: A Simple Probability-Based Approach

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Abstract.—Precautionary fishery management requires that a distinction be made between target and limit reference points. We present a simple probability framework for deriving a target reference point for the fishing mortality rate (F) or biomass (B) from the corresponding limit reference point. Our framework is a generalization of one devised previously by Caddy and McGarvey (1996). Both methods require an a priori management decision on the allowable probability of exceeding the limit reference point; our method removes a major assumption by accounting for the uncertainty in the limit reference point. We present the theory underlying the method, an algorithm for solution, and examples of its application. The new procedure, like the old, requires an estimate of the implementation uncertainty expected in the following year's management, an estimate that might be obtained by a review of the effectiveness of past management actions. Either method can be implemented easily on a modern desktop computer. Our generalized framework is more complete, and we believe that it has wide applicability in the use of fishery reference points or, for that matter, in other conservation applications that strive for resource sustainability.

In recent years, precautionary management of fisheries (e.g., FAO 1995) has become well established. In defining and implementing precautionary management, the concepts of limit reference

point and target reference point have been found useful by scientists and managers (Smith et al. 1993; Mace 1994; Caddy 1998). These concepts were promoted by the United Nations Conference on Straddling Fish Stocks and Highly Migratory Fish Stocks (United Nations 1995) and the United Nations Code of Conduct for Responsible Fishing (Caddy and Mahon 1995). In simple terms, a limit reference point (LRP) reflects the perceived max-

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Table 1.—Abbreviations and mathematical symbols.

Symbol	Description
CM	Method of Caddy and McGarvey (1996) for finding a target reference point from a precise limit reference point
CV	Coefficient of variation (SD/mean)
REPAST	Our method (an extension of CM) for finding a target reference point from an imprecise limit reference point
LRP	Limit reference point (in general)
MSY	Maximum sustainable yield
TRP	Target reference point (in general)
pdf	Probability density function
cdf	Cumulative distribution function (integral of pdf)
F	Instantaneous rate of fishing mortality
B	Biomass of stock
F_λ, B_λ	Value of F or B chosen to implement an LRP
F_τ, B_τ	Value of F or B chosen to implement a TRP
F_{now}, B_{now}	Estimated value of F or B at the close of the last observed period (typically, the year just ended)
F_{next}, B_{next}	Realized value of F or B in the management period (typically, the next year)
F_{MSY}, B_{MSY}	Value of F or B at which MSY can be realized
R_λ	LRP in F or B expressed relative to F_{now} or B_{now}
R_τ	TRP in F or B expressed relative to F_{now} or B_{now}
R_{next}	Realized value of F or B in the management period expressed relative to F_{now} or B_{now}
P^*	Allowable probability of exceeding an LRP in the next management period
ϕ	Dummy variable used in double integration
$\sigma_\lambda, \sigma_{F_{next}}$	Standard errors of LRP and TRP

imum degree of safe exploitation for a stock. It is implicit that an LRP should rarely be exceeded (Mace and Sissenwine 2002). Depending on the assessment and management techniques in use, an LRP can be expressed in terms of fishing mortality rate (F), stock biomass (B), spawning-stock biomass (SSB), or other metric of exploitation rate or stock abundance. (All of the symbols and abbreviations used in this paper are listed in Table 1.) A target reference point (TRP) uses the same metric as the corresponding LRP and defines the degree of exploitation aimed for under management. When reference points are measured in F, the preceding definitions imply that the TRP is no greater than the LRP; when reference points are measured in B or SSB, they imply that the TRP equals or exceeds the LRP. Stock assessment and management are uncertain, and the difference between the TRP and the LRP constitutes a margin of safety that prevents frequent occurrences of exploitation beyond the LRP and thus promotes sustainability (Mace 2001).

In the United States, recent changes brought about by the Sustainable Fisheries Act have introduced a precautionary approach to fishery management at the federal level (U.S. Office of the Federal Register 1998). Technical guidelines (Restrepo et al. 1998) issued to implement that act suggest methods for computing reference points in B and F and corresponding control rules. Thus, the use of reference points in U.S. marine fishery management has become widespread and is likely to continue. In U.S. technical and regulatory documents (e.g., Restrepo et al. 1998), LRPs are often called thresholds.

When establishing reference points, how one chooses among competing models is a very broad question, one whose answer will depend on the nature of the resource and the fishery. Here we explore a different, but nonetheless important, question: Given an LRP, how can the corresponding TRP be computed? Implicit in that question is the assumption that management can decide on a suitable LRP (e.g., F_{MSY} [the fishing mortality rate associated with the maximum sustainable yield] or a minimum spawning-stock threshold) and that suitable assessment models can provide a working estimate of its value.

The Caddy–McGarvey Framework for Setting a TRP

One approach to computing TRP corresponding to a specified LRP was provided by Caddy and McGarvey (1996), who based their argument on simple statistical theory. They assumed that the TRP is the central tendency of a probability density function (pdf) that describes the uncertain outcome of a given set of management actions. They then showed that, if the shape of the pdf is known, the TRP can be calculated from an acceptable probability (P^*) of exceeding the LRP.

Caddy and McGarvey (1996) developed the mathematical representation of their methodology (which we denote as CM) using the fishing mortality rate as the management control variable. Ambiguously, they used the symbol F_{now} to refer to both the target reference point in F, which is a fixed number, and the current fishing mortality rate, which is a quantity estimated with uncertainty. Here we distinguish those two concepts by using F_τ to represent the target reference point and F_{next} for the realized fishing mortality rate in the management period, typically the next year. In our notation, the CM framework is represented as

$$\Pr(F_{next} > F_\lambda) = \int_{F_\lambda}^{\infty} \text{pdf}_{F_{next}}(F)\, dF = P^*, \quad (1)$$

where $\Pr(x)$ is the probability of condition x, F_λ is the limit reference point in F, and $\text{pdf}_{F_{next}}(F)$ is the
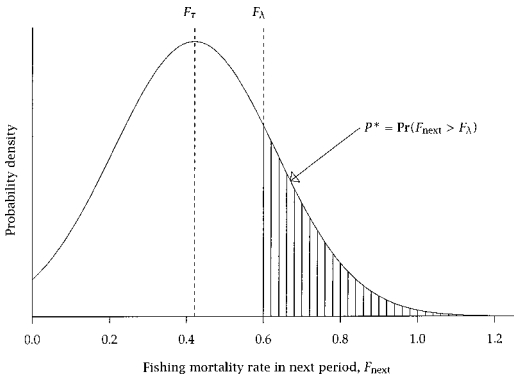
FIGURE 1.—Probability density function illustrating the Caddy-McGarvey (CM) procedure. The figure shows the relationships between the limit reference point (F_λ), the assumed variability of the fishing mortality rate in the next period (F_{next}), the allowable probability that $F_{\text{next}} > F_\lambda$ ($P*$), and the resulting target reference point (F_τ).

pdf of F_{next} evaluated at F. Caddy and McGarvey's assumption that $\text{pdf}_{F_{\text{next}}}(F)$ is centered on the TRP implies a belief that implementation of the TRP, although imprecise, is accurate. Consequently, when F_τ is increased or decreased, $\text{Pr}(F_{\text{next}} > F_\lambda)$ increases or decreases accordingly so that some particular value of F_τ provides the desired probability $P*$ (Figure 1).

Solution of the CM Framework for TRP

The solution of equation (1) for F_τ is completed in two steps. First, one must specify the pdf of F_{next} and estimate or assume its parameters. Second, one must use a solution algorithm to find the value of F_τ corresponding to the desired $P*$. If the pdf is normal or lognormal, then its location parameter (mean or median) will be based on F_τ and only its dispersion parameter (SD of F_{next} around F_τ) will remain to be specified. For example, if F_{next} is normally distributed with mean F_τ and standard deviation $\sigma_{F_{\text{next}}}$, equation (1) becomes

$$\text{Pr}(F_{\text{next}} > F_\lambda)$$
$$= \int_{F_\lambda}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_{F_{\text{next}}}} \exp\left[-\frac{(F - F_\tau)^2}{2\sigma_{F_{\text{next}}}^2}\right] dF = P*. \quad (2)$$

In the CM framework, $P*$ has been established by managers; F_λ is assumed to be available from assessment results; and $\sigma_{F_{\text{next}}}$ is assumed estimable by some systematic or ad hoc method, as discussed later. The only unknown is the value of F_τ that will make the equation true.

To effect the second step of the solution, equation (2) can be solved for F_τ by evaluating the integral at successive trial values of F_τ until a value that satisfies the equation is found. The trial values are chosen in a systematic way, such as by the bisection method (Gill et al. 1981). Although the integral has no explicit solution, numerical methods of integration are easily within the capability of modern computers.

Because the integral in equation (2) must be evaluated numerically, finding the solution value of F_τ is an iterative process. A simpler approach is to use an approximation of the inverse-normal function Z^{-1} (e.g., Adams 1969). For a random variable $x \sim N(0, 1)$, the inverse-normal function is defined as

$$Z^{-1}(\pi) \equiv z \text{ such that Pr } (x < z) = \pi. \quad (3)$$

Given the ability to compute the inverse-normal function, it is possible to compute the expected TRP directly as

$$F_\tau = \frac{F_\lambda}{1 + \text{CV}_{F_{\text{next}}} \cdot Z^{-1}(1 - P*)}. \quad (4)$$

Similarly, if F_{next} is lognormally distributed, one may compute the median TRP as

$$F_\tau = \frac{F_\lambda}{\exp[\sigma_{F_{\text{next}}} \cdot Z^{-1}(1 - P*)]}. \quad (5)$$

In equation (4), we use the coefficient of variation (CV) of the TRP rather than its SD, but of course the two are interchangeable by the relationship CV = SD/mean. In a specific example of this approach, Caddy and McGarvey (1996) gave numerical approximations from which F_τ could be computed, assuming normal uncertainty in F_{next} and given $\sigma_{F_{\text{next}}}$, F_λ, and $P*$.

Generalization of CM for Uncertainty in the Limit Reference Point

The CM approach is attractive for several reasons. Chief among them are that it recognizes uncertainty in the TRP, it is conceptually simple and thus easily communicated, and it is based on an explicit probability framework rather than completely ad hoc reasoning. Importantly, specification of $P*$ by managers emphasizes the nonscientific dimension of setting a target. The most obvious limitation of the CM framework is that it assumes zero variability in the LRP. That assumption is not realistic because any estimate of a limit reference point (whether measured in *F, B,* or another metric) is derived from imprecise data and thus is imprecise itself. It is not difficult to gen-

eralize the preceding framework to account for imprecision in F_λ as well as in F_τ. The more general form of equation (1) is

$$\Pr(F_{\text{next}} > F_\lambda)$$

$$= \int_{-\infty}^{\infty} \Pr(F_{\text{next}} > F) \cdot \Pr(F_\lambda = F) \, dF, \quad (6)$$

or equivalently

$$\Pr(F_{\text{next}} > F_\lambda)$$

$$= \int_{-\infty}^{\infty} [1 - \text{cdf}_{F_{\text{next}}}(F)] \cdot \text{pdf}_{F_\lambda}(F) \, dF, \quad (7)$$

where evaluation of the cumulative distribution function $\text{cdf}_{F_{\text{next}}}(F)$ may require integration or numerical approximation. Equations (6) and (7) can be interpreted as summing weighted averages of the CM method across all possible values of F_λ, the statistical weights being the relative probabilities of observing those values of F_λ. The generalization assumes that the uncertainty in F_{next} and that in F_λ are independent, an assumption that is relaxed later.

We illustrate our generalized framework using two hypothetical examples that differ only in the location parameter (mean or median) of the TRP (Figures 2, 3). The figures parallel equation (7); in each figure, the uppermost plot shows $1 - \text{cdf}_{F_{\text{next}}}(F)$, the center plot shows $\text{pdf}_{F_\lambda}(F)$, and the lowermost plot shows the product of the two, that is, the full integrand in equation (7). The area under the lowermost curve is $\Pr(F_{\text{next}} > F_\lambda)$. Comparison of the two figures reveals that, as expected, the probability of exceeding the LRP becomes lower as the TRP is reduced from $F_\tau = 0.4$ (Figure 2) to $F_\tau = 0.3$ (Figure 3). A similar reduction would occur if the LRP were made higher or the SD of either the TRP or LRP were reduced.

When statistical distributions for F_{next} and F_λ are known or estimated, equation (7) can be written in a more explicit form. Assuming normal distributions, for example, the new equation is

$$\Pr(F_{\text{next}} > F_\lambda)$$

$$= \frac{1}{2\pi\sigma_{F_{\text{next}}}\sigma_\lambda} \int_{-\infty}^{\infty} \left\{ \int_{F}^{\infty} \exp\left[\frac{(\phi - F_\tau)^2}{2\sigma_{F_{\text{next}}}^2}\right] d\phi \right\} \quad (8)$$

$$\times \exp\left[\frac{(F - F_\lambda)^2}{2\sigma_\lambda^2}\right] dF$$

where $\sigma_{F_{\text{next}}}$ is the standard error of F_{next}, σ_λ is the
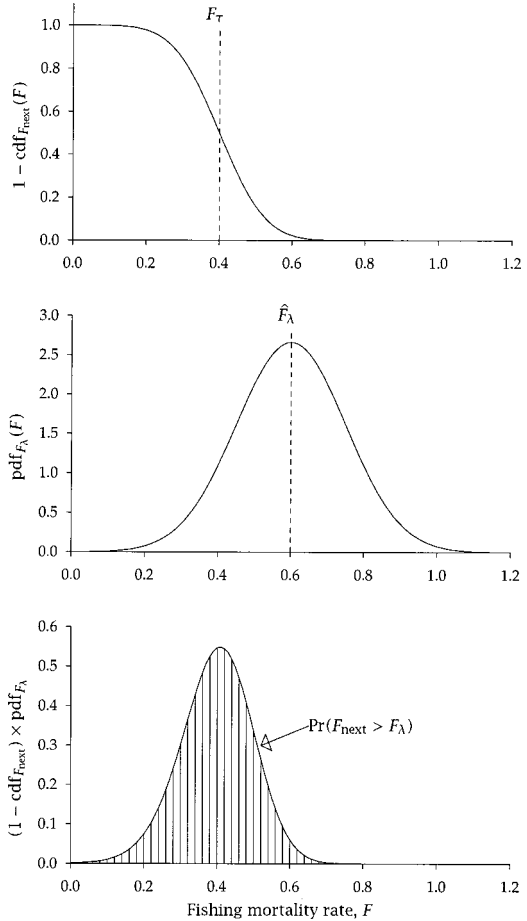

FIGURE 2.—Diagram 1 of the generalized procedure for computing a TRP, showing the relationships between the terms in equation (7) and the fishing mortality rate (F). The abbreviation pdf stands for probability density function, the abbreviation cdf for cumulative distribution function. In this diagram, the coefficient of variation (CV; defined as $100 \cdot$ SD/mean) is 25% for both the target reference point and the limit reference point.

standard error of F_λ, and ϕ is a dummy variable. Given a value of P^* chosen by managers and estimates of $\sigma_{F_{\text{next}}}$, σ_λ, and F_λ, equation (8) can be solved for F_τ. The double integral here can be more time-consuming to compute than the single integral in equation (2), and the possibility of a direct solution based on an approximation for Z^{-1} is no longer available (although approximation of Z^{-1} for the inner integral can be used to speed the computations). The only additional data requirement of the generalized framework is an estimate of σ_λ.

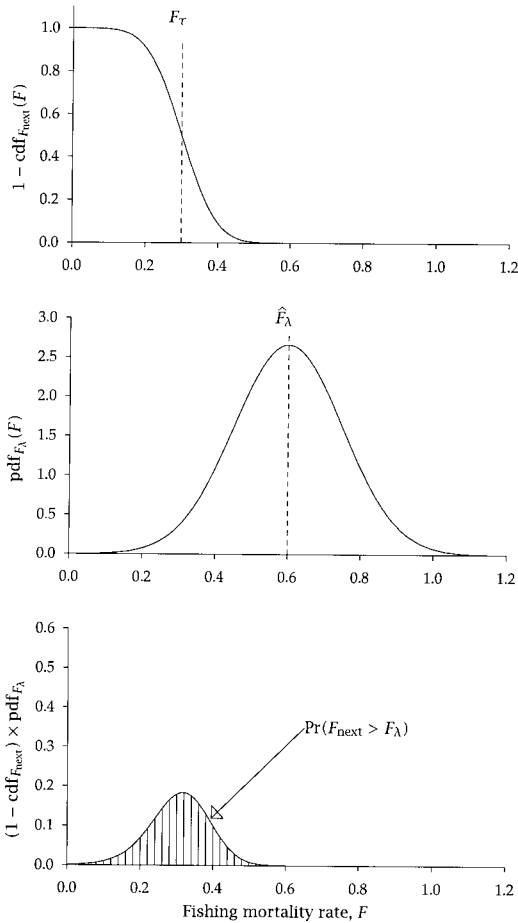With minor adjustments, the same approach can be used for lognormally distributed uncertainty. It

FIGURE 3.—Diagram 2 of the generalized procedure for computing a TRP, which is the same as Figure 2 except that the value of the target reference point has been reduced. Note the reduced probability that $F_{next} > F_{\lambda}$.

can also be adapted to other continuous distributions as long as their density and distribution functions can be characterized from the information at hand and then evaluated analytically or numerically.

For simplicity, we have assumed above that the uncertainties in F_{next} and F_{λ} are independent. That assumption is convenient but not necessary. It is possible that the two quantities are correlated, as they are estimated from the same data and assessment framework. In that case, one could estimate the joint probability density of F_{next} and F_{λ} (e.g., a bivariate normal distribution) and integrate it over the appropriate region. We suspect, however, that in practice the correlation between the two quantities will be low owing to the large imple-

mentation uncertainty in F_{next}, which is quite separate from the estimation uncertainty involved in finding F_{λ}, and that as a consequence equation (7) will be applicable. We next present an approach that further reduces the possibility of correlation between F_{next} and F_{λ}.

Ratio-Extended Probability Approach to Setting Targets (REPAST)

From the generalized framework described above, we now develop a variant that appears well suited to application in fishery management while avoiding the main source of correlation between F_{next} and F_{λ}. Because this variant is based on dimensionless quantities that can be written as ratios, we call it REPAST, for ratio-extended probability approach to setting targets. The REPAST framework was developed while considering properties of F_{MSY} as estimated from surplus-production models, and we explain it in that context. Nonetheless, we believe that analysts will find it applicable to other LRPs and assessment procedures as well.

As in many quantitative problems, progress can be made by replacing important variables with related dimensionless (scale-independent) quantities (Barenblatt 1996). In fishery science, the scale-independent approach has been used, for example, in developing the concept of spawning potential ratio (Goodyear 1993). Estimates of population states B_t and F_t at time t from a surplus-production model are more precise when expressed as dimensionless proportions of B_{MSY} and F_{MSY}, respectively, than in specific units of mass and time^{-1} (Prager 1994). In dimensionless form, the estimates no longer incorporate information on the catchability coefficient q, which is often poorly estimated. Indeed, determining the exact scale of a population (equivalent to determining q) is one of the most difficult problems in fish population dynamics (Smith 1994). An additional reason for preferring the dimensionless estimates is that the effects of bias and error in the sampling program will tend to cancel one another out. For example, if only a consistent fraction of the population is sampled, the usual (scaled) estimate of B_t will be biased, but the dimensionless estimate will be unaffected.

It follows that the limit reference point F_{λ}, which in this context is equated to F_{MSY}, can be expressed with greater precision as a ratio to the current (final-year) fishing mortality rate than it can be in absolute terms. We designate that ratio R_{λ}, defined here as F_{MSY}/F_{now} and more generally as F_{λ}/F_{now}.

The quantity R_λ is a dimensionless LRP that is known with statistical error. It should be a routine matter to estimate it from an assessment model and almost as routine to obtain an estimate of its standard error or coefficient of variation.

The management of fishing mortality rate is also usually effected in a relative sense (as is management based on total stock biomass). By that we mean that the target fishing mortality rate in the next period, F_τ, is generally set by proportional adjustment to the current fishing mortality rate F_{now} rather than by some totally new analysis of fishing power, fishing effort rate, and so forth. For the desired adjustment, we use the notation R_τ, defined such that $F_\tau = R_\tau \cdot F_{now}$. Thus the quantity R_τ is a dimensionless TRP taking the form of a multiplier that will be implemented with statistical error. We assume as before that the multiplier actually achieved, R_{next}, is uncertain and can be described by a pdf centered on the desired TRP, R_τ. Despite the transformation into dimensionless quantities, the method of attack remains the same. Computing the probability that $F_{next} > F_\lambda$ is essentially the same as computing the probability that $R_{next} > R_\lambda$. Equation (7) becomes

$$\Pr(R_{next} > R_\lambda)$$
$$= \int_{-\infty}^{\infty} [1 - \text{cdf}_{R_{next}}(R)] \cdot \text{pdf}_{R_\lambda}(R)\, dR, \quad (9)$$

which can be solved for the value of R_τ that will produce the allowable probability P^* that $R_{next} > R_\lambda$. The solution is possible when the pdfs of R_{next} and R_λ are known or can be estimated. As with the CM method, any distributions can be specified, including empirical ones.

Although conceptually REPAST is almost identical to our non-ratio-based generalization, there are two advantages of using dimensionless reference points. First, the uncertainty in the dimensionless quantities should generally be less than that in the original reference points because the problem of scaling the population is avoided. Second, the correlation between uncertainty in achieving the target and that in estimating the LRP is greatly reduced. As mentioned above, even the scaled quantities F_{next} and F_λ should be uncorrelated because their major uncertainties stem from independent processes; the uncertainty in F_{next} largely reflects imperfect implementation of regulations, while the uncertainty in F_λ reflects estimation and sampling error. In practice, however, errors in the two quantities will be correlated if

there is an overall bias to the sampling regime. In contrast, the uncertainty in the dimensionless quantity R_{next} depends only on implementation, not on sampling, and thus R_{next} will be uncorrelated with R_λ—except, perhaps, to the degree that compliance with regulations is correlated with their severity.

Whether the calculations are done in terms of scaled or dimensionless reference points, the best method of quantifying implementation uncertainty is not obvious. An ad hoc approach might be to postulate a provisional value by assuming a CV of R_{next}. A more empirical approach would be to estimate uncertainty from data on the past performance of the fishery. By analyzing the past intended management of F and the results obtained, it should be possible to estimate the CV of R_{next}. An example of data-based modeling of such partial management control of a wild population is provided by Johnson et al. (1997).

Examples

Three examples follow. The first demonstrates the similarities and differences between the CM procedure and REPAST; the second and third apply REPAST to swordfish *Xiphias gladius* in the north Atlantic Ocean. In these examples, TRPs in terms of fishing mortality rate and biomass are based on estimates of the CV of F_{MSY} from a surplus-production model. The applications to swordfish are intended strictly as examples and do not provide definitive information on that stock.

Example 1: Comparison with the Caddy–McGarvey Procedure

In this example, we take a case given in Caddy and McGarvey (1996) and recompute it using the REPAST generalization. The example is based on three assumptions: (1) The dimensionless limit reference point $R_\lambda = F_\lambda/F_{now} = 0.6$. That is, the present fishing mortality rate is higher than the established LRP by the factor $1/0.6 \sim 1.67$. (2) Implementation of the dimensionless target reference point R_τ is uncertain and characterized by the CV of R_{next}. (3) To make our numerical results directly comparable those of Caddy and McGarvey (1996), we assume that $F_{now} = 1.0$/year. In that case, by definition $F_\lambda = R_\lambda$/year and $F_\tau = R_\tau$/year.

If R_λ is specified as a point value, the example is the same as that of Caddy and McGarvey (1996). The resulting values of the target reference point R_τ are given in Table 2 for a range of values of P^* and CV of R_{next}. That table is more detailed than Table 1 of Caddy and McGarvey (1996) and also

TABLE 2.—Example with a unitless limit reference point R_λ (= F_{MSY}/F_{now} = 0.6), using the CM method (R_λ assumed precise). Values are those of a unitless target reference point R_τ that provide specified probabilities P^* of exceeding the LRP in the next period as a function of the CV of R_{next}. Abbreviations and symbols are defined in Table 1.

	CV of R_{next}									
P^*	0.05	0.10	0.15	0.20	0.25	0.33	0.50	0.66	0.80	1.00
	Normal distribution of uncertainty (values are means)									
50%	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60
40%	0.59	0.59	0.58	0.57	0.56	0.55	0.53	0.51	0.50	0.48
35%	0.59	0.58	0.57	0.56	0.55	0.53	0.50	0.48	0.46	0.43
30%	0.58	0.57	0.56	0.54	0.53	0.51	0.48	0.45	0.42	0.39
25%	0.58	0.56	0.54	0.53	0.51	0.49	0.45	0.42	0.39	0.36
20%	0.58	0.55	0.53	0.51	0.50	0.47	0.42	0.39	0.36	0.33
15%	0.57	0.54	0.52	0.50	0.48	0.45	0.40	0.36	0.33	0.29
10%	0.56	0.53	0.50	0.48	0.45	0.42	0.37	0.33	0.30	0.26
5%	0.55	0.52	0.48	0.45	0.43	0.39	0.33	0.29	0.26	0.23
	Lognormal distribution of uncertainty (values are medians)									
50%	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60
40%	0.59	0.59	0.58	0.57	0.56	0.55	0.53	0.52	0.50	0.49
35%	0.59	0.58	0.57	0.56	0.55	0.53	0.50	0.48	0.46	0.44
30%	0.58	0.57	0.55	0.54	0.53	0.51	0.47	0.44	0.41	0.39
25%	0.58	0.56	0.54	0.52	0.51	0.48	0.44	0.40	0.37	0.34
20%	0.58	0.55	0.53	0.51	0.49	0.46	0.40	0.36	0.33	0.30
15%	0.57	0.54	0.51	0.49	0.46	0.43	0.37	0.32	0.29	0.25
10%	0.56	0.53	0.50	0.47	0.44	0.40	0.33	0.28	0.24	0.21
5%	0.55	0.51	0.47	0.43	0.40	0.35	0.28	0.22	0.19	0.15

TABLE 3.—Example with a unitless limit reference point R_λ (= F_{MSY}/F_{now} = 0.6), using the REPAST method with a 25% CV of R_λ. Values are those of a unitless target reference point R_τ that provide specified probabilities P^* of exceeding the LRP in the next period as a function of the CV of R_{next}. Abbreviations and symbols are defined in Table 1.

	CV of R_{next}									
P^*	0.05	0.10	0.15	0.20	0.25	0.33	0.50	0.66	0.80	1.00
	Normal distribution of uncertainty (values are means)									
50%	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60
40%	0.56	0.56	0.56	0.55	0.55	0.54	0.52	0.51	0.49	0.47
35%	0.54	0.54	0.53	0.53	0.52	0.51	0.49	0.47	0.45	0.43
30%	0.52	0.52	0.51	0.51	0.50	0.48	0.46	0.43	0.41	0.38
25%	0.50	0.49	0.49	0.48	0.47	0.46	0.42	0.40	0.37	0.35
20%	0.47	0.47	0.46	0.45	0.44	0.43	0.39	0.36	0.34	0.31
15%	0.44	0.44	0.43	0.42	0.41	0.39	0.36	0.33	0.30	0.28
10%	0.41	0.40	0.39	0.38	0.37	0.36	0.32	0.29	0.27	0.24
5%	0.35	0.35	0.34	0.33	0.32	0.30	0.27	0.24	0.22	0.20
	Lognormal distribution of uncertainty (values are medians)									
50%	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60
40%	0.56	0.56	0.56	0.55	0.55	0.54	0.52	0.51	0.50	0.48
35%	0.54	0.54	0.54	0.53	0.52	0.51	0.49	0.47	0.45	0.43
30%	0.53	0.52	0.52	0.51	0.50	0.49	0.45	0.43	0.41	0.38
25%	0.51	0.50	0.49	0.48	0.47	0.46	0.42	0.39	0.36	0.33
20%	0.49	0.48	0.47	0.46	0.45	0.43	0.38	0.35	0.32	0.29
15%	0.46	0.46	0.45	0.43	0.42	0.39	0.35	0.31	0.28	0.24
10%	0.43	0.43	0.41	0.40	0.38	0.36	0.30	0.26	0.23	0.20
5%	0.40	0.39	0.37	0.36	0.34	0.31	0.25	0.21	0.18	0.14

corrects an apparent error, namely, their value of F_τ = −0.04/year for P^* = 0.05 and CV = 1.00, but otherwise it displays the same values.

We next assume, more realistically, that the dimensionless LRP R_λ is estimated with error (CV = 0.25) and use REPAST to compute R_τ accord-ingly. The result is a slightly lower target at each combination of P^* and CV of R_{next} (Table 3). The values of R_τ are perhaps most informative when presented as a contour plot (Figure 4a, b), as are the differences between procedures (Figure 4c, d). Those differences are largest when the CV of R_{next}
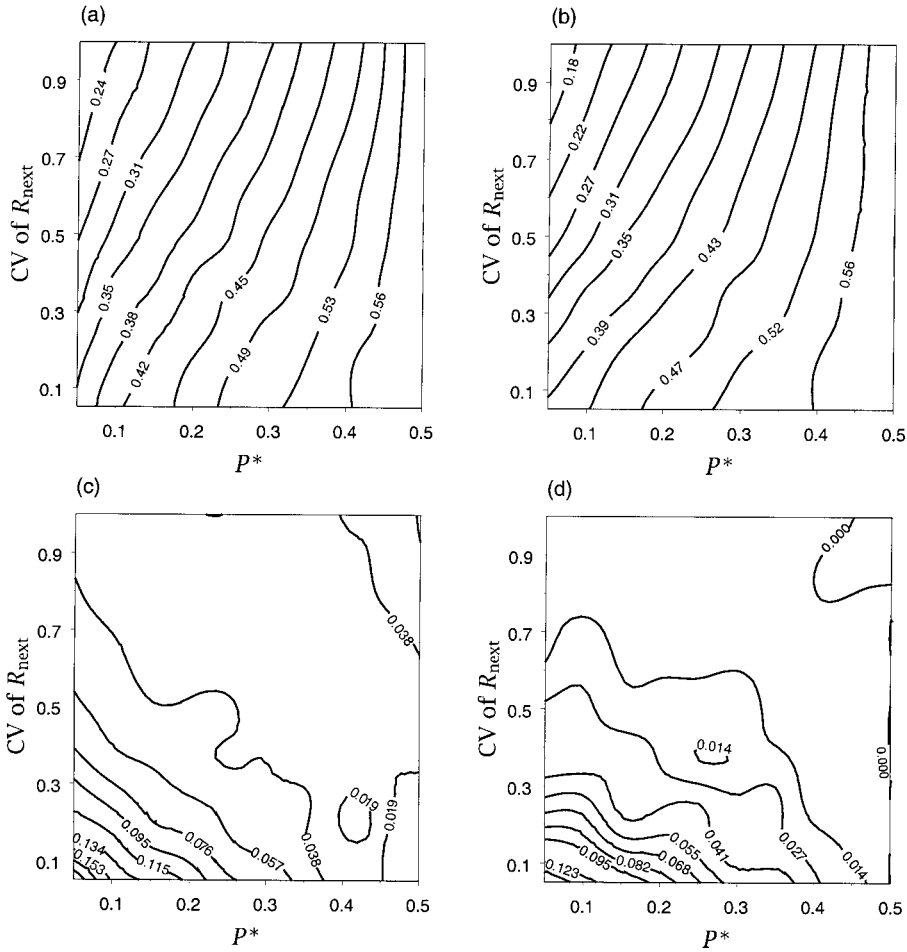
FIGURE 4.—Panels **(a)** and **(b)** show the contours of R_τ, here a dimensionless target reference point in F expressed as a proportion of the current F. These contours depend on the allowable probability (P^*) of exceeding R_λ, the dimensionless limit reference point in F (x-axis); the value of R_λ; the estimated mean and CV of R_λ (here assumed to be 0.6 and 25%, respectively); and the CV of R_{next}, the dimensionless value of F actually achieved (on the y-axis). Panels **(c)** and **(d)** show the increases in R_τ from using the CM procedure, which disregards uncertainty in the estimation of R_λ.

is low because then the uncertainty in the limit reference point R_λ becomes more important.

Example 2: North Atlantic Swordfish

In this example, we apply REPAST to results from a surplus-production model, using F_{MSY} as the limit reference point (Mace 2001). Prager (2002) examined several aspects of production modeling of swordfish in the north Atlantic Ocean based on catch and relative-abundance data for 1950–1998. Prager's analysis using a trimmed least-squares fit of the generalized production model provides an estimate of $R_\lambda = F_{MSY}/F_{1998} = 0.814$. We repeated that production-model analy-

sis, adding a bootstrap, as in Prager (1994), to generate an empirical sampling distribution of R_λ (Figure 5). That distribution implies a CV around R_λ of 0.263. Because the normal distribution appears to be a good approximation (Figure 5), we assume normality for this example.

Using $R_\lambda = 0.814$ and CV = 0.263, we solved equation (9) for the TRP over a range of values of P^* and CV of R_{next} (Figure 6). For example, at $P^* = 20\%$ and CV = 0.25, $R_\tau = 0.60$, meaning that the appropriate target fishing mortality rate in the next period is 60% of the current F. In general, with a lower P^* or higher CV of R_{next}, the value of R_τ decreases.
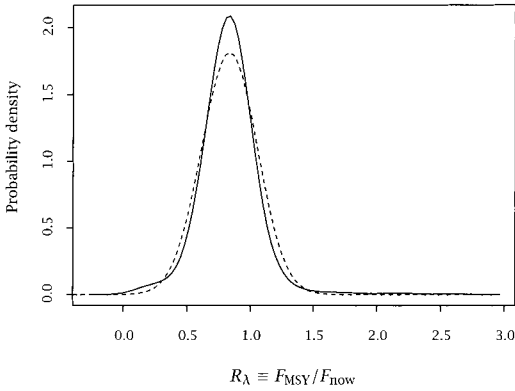
FIGURE 5.—Solid line: empirical probability density of $R_\lambda = F_{MSY}/F_{now}$ from bootstrap fit of generalized production model to trimmed data on swordfish in north Atlantic Ocean. Dashed line: normal probability density with equal mean and SD, shown for comparison. Abbreviations and symbols are defined in Table 1.

Example 3: Reference Point in Biomass

This example is based on the same surplus-production model of swordfish but differs by considering reference points in stock biomass rather than fishing mortality rate. For the sake of the example, we assumed that the LRP in biomass, B_λ, equals $0.75B_{MSY}$, the value suggested as a possible minimum-stock-size threshold in Restrepo et al. (1998). We use the same notation as before, but here $R_\lambda = B_\lambda/B_{now}$ and $B_\tau = R_\tau \cdot B_{now}$, the distinction between dimensionless reference points in biomass and those in fishing mortality rate being clear from the context. The point estimate of R_λ in biomass from the production model is 1.06, while in this case the bootstrap distribution of R_λ is characterized by a CV of 0.189. Again, the distribution appeared to be close to normal (Figure 7), so the normal assumption was used in applying REPAST. Because of the change in reference points from F to B, the following replaces equation (9):

$$\Pr(R_{next} < R_\lambda)$$

$$= \int_{-\infty}^{\infty} [\text{cdf}_{R_{next}}(R)] \cdot \text{pdf}_{R_\lambda}(R) \, dR. \qquad (10)$$

The important point is the reversed inequality on the left-hand side of the new equation, reflecting the fact that TRP ≥ LRP when the reference is biomass.

As with the previous example, the results are presented as a contour plot (Figure 8). We chose the same point to exemplify the results, namely, $P^* = 20\%$ and CV = 0.25. Given the stated LRP,
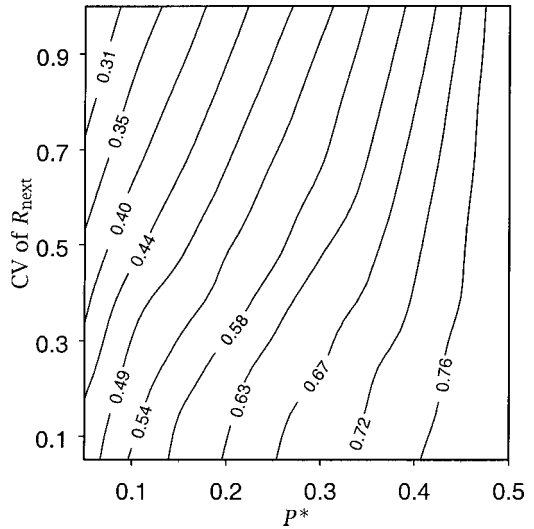

FIGURE 6.—Contours of R_τ (here a dimensionless target reference point in F expressed as a proportion of the current value of F) for North Atlantic swordfish calculated via our ratio-extended probability approach to setting targets (REPAST) with normal uncertainties ($R_\lambda = 0.814$ with CV = 0.263). These computations are for purposes of illustration only.

the target must be at least 140% of B_{now} (Figure 8). In general, R_τ increases with lower values of P^* or higher values of the CV of R_{next}.

Discussion

We have described a simple framework for computing target reference points from limit reference points, a framework based on the work of Caddy
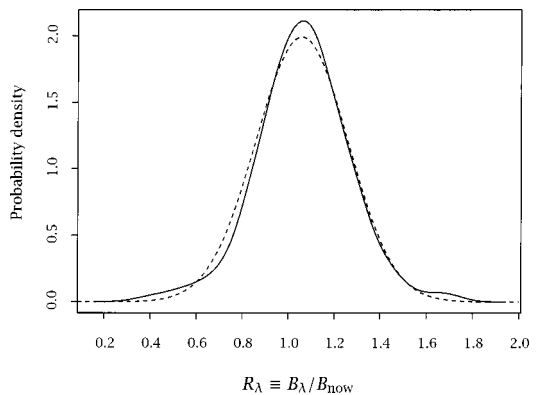

FIGURE 7.—Solid line: empirical probability density of $R_\lambda = B_\lambda/B_{now}$ from bootstrap fit of generalized production model to trimmed data on swordfish in north Atlantic Ocean. Dashed line: normal probability density with equal mean and SD, shown for comparison. Abbreviations and symbols are defined in Table 1.
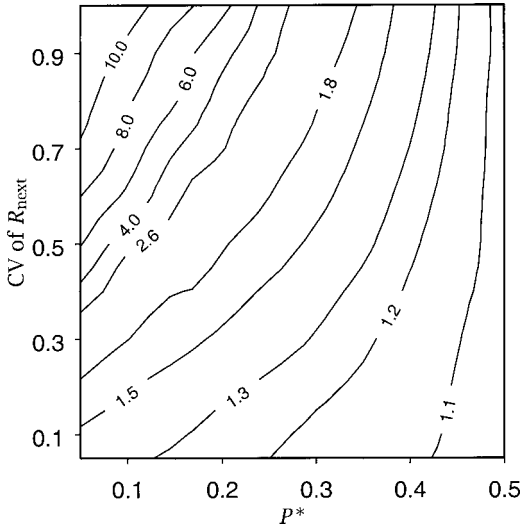
FIGURE 8.—Contours of R_τ (here a dimensionless target reference point in biomass [B] expressed as a proportion of the current value of B) for North Atlantic swordfish calculated via our ratio-extended probability approach to setting targets (REPAST) with normal uncertainties ($R_\lambda = 1.06$ with CV = 0.189). These computations are for purposes of illustration only.

and McGarvey (1996) but incorporating two major extensions. First, it allows for uncertainty in the estimation of the limit reference point and thus provides a more accurate picture of reality than the old procedure. Because the magnitude of that uncertainty can be estimated routinely by modern assessment models, the added data burden of this extension is small. Because that uncertainty is not assumed to be negligible, the targets derived from the new procedure are somewhat more conservative than those from the old procedure.

The second refinement consists of casting our framework in terms of dimensionless indicators of stock status. While that approach may seem more complex, we believe that it has advantages. It recognizes that management is usually applied by adjusting present fishing regimes; it takes advantage of the cancellation of sampling errors in estimated quantities, thus improving precision; and it reduces concern about the possible correlation of the errors in the LRP and the realized target. By increasing precision, the use of dimensionless quantities may allow higher exploitation rates than a similar procedure using scaled values.

The disadvantages of our new procedure are that it is slightly more difficult conceptually than the original and that the computations are a bit more complex. We hope that our explanations have mit-

igated the first disadvantage and that free availability of software for the procedure (explained below) will mitigate the second.

Statistical Issues

A potential concern about dimensionless reference points is that, as ratios, they may have undesirable statistical properties. Indeed, the ratio of a constant to a normally distributed random variable (for example) has a U-shaped distribution that would be unsuitable for use with REPAST. Furthermore, it is frequently recommended that proportions, which are another type of ratio, be transformed before analysis (Snedecor and Cochran 1980). In contrast, ratio estimates are recommended as being more precise than estimates of individual quantities when the correlation between the numerator and denominator is high (Snedecor and Cochran 1980), as it should be in the ratios $R_\lambda = F_\lambda/F_{\text{now}}$ and $R_\tau = F_\tau/F_{\text{now}}$. For example, ratio estimators are recommended by Snedecor and Cochran (1980: 456) for estimating relative population sizes over time, a use reportedly introduced by Laplace in the early 1800s (Rao 1986). Thus, the use of dimensionless quantities (ratios) in REPAST appears statistically well founded. In addition, we note that the basic equations of surplus-production models involve not biomass itself but biomass as a dimensionless ratio to carrying capacity (e.g., Fletcher 1978; Prager 1994; Quinn and Deriso 1999). For a fixed production model shape (i.e., one with a fixed exponent in the generalized production equation), that is fundamentally the same dimensionless approach we have taken. In that sense, dimensionless estimates are more fundamental products of production models than the corresponding scaled estimates. As noted above, scaling is a major source of uncertainty even in more complex population models (Smith 1994).

When the CV of F_{next} is relatively large, as in the first example, and the variability is assumed to be distributed normally, a noticeable portion of the distribution of F_{next} may lie below zero. There are two strategies for responding to this situation. One can either assume that all negative values of F_{next} are equivalent to $F = 0$, or one can renormalize the portion of the density function over the range $0 < F < \infty$ so that its integral is unity. The value of the TRP provided by our methods will depend on which strategy is used. In our examples we used the first strategy, but we have no strong preference for one or the other. We do think that once a choice is made it should be maintained in

future assessments of the stock. We view the value of our methods not as providing TRPs that precisely match the chosen $P*$ but as providing repeatable, objective, statistically based TRPs that approximate the chosen $P*$. If neither strategy is acceptable, the entire issue can be avoided by using the equations for lognormally distributed uncertainty.

Reference Points in Biomass

The examples show a structural difference between reference points in biomass and those in fishing mortality rate. In the latter, the LRP is often set at an estimate of (or proxy for) F_{MSY}, and for that reason the nearer the applied fishing mortality is to the LRP the higher the sustainable yield. In that sense, the REPAST procedure provides an optimal fishing mortality rate within the constraints of $P*$. The LRP in biomass, in contrast, is usually set lower than B_{MSY}. Driving stock levels as near as possible to such an LRP would be a risk-prone approach that also reduced the sustainable yield. Therefore, we suggest that when using REPAST to compute reference points in biomass the TRP be set to the computed value only if it would result in $B_\tau \geq B_{MSY}$ and to B_{MSY} otherwise.

The application of the CM approach or our generalizations of it to biomass-based TRPs is subtly different from that to F-based TRPs in that biomass is not directly controlled by managers. Except where stocking is used, managers can increase the population only indirectly (by implementing rules to reduce F) and thus provide a larger stock biomass at some future time. This implies an added source of uncertainty, namely, that associated with the time it takes for the biomass to increase to the reference point. In principle, this aspect of the problem could be made transparent to managers through a model of the added uncertainty; such a model would describe the probability density of achieving B_λ (or R_λ) within a prescribed time.

Setting P*

As noted by Shotton (1993), the probability $P*$ of exceeding the limit reference point acceptable to managers will depend on their aversion to risk. If they are relatively risk prone, for example, they may choose $P* > 0.1$ for use with these methods. If they are more risk averse, they will tend to choose a lower probability of exceeding the LRP. Whatever their preferences for risk, $P*$ is not an output (as it is in many approaches) but an input that must be specified a priori to arrive at a target. We believe that it is important that the specification

of this probability be recognized for the political (i.e., management) decision it is and neither relegated to science, which cannot answer it, nor swept under the rug. Thus, we view the need to set $P*$ explicitly and a priori as a strength of the methods described here.

It is possible, nonetheless, that science can aid managers in determining a value of $P*$ that is optimal in some sense. Formal risk analysis provides a framework for quantifying risk, defined in that context as the mathematical expectation of loss from a policy. Thus, computer simulation of the stock's biology, management, and fishery could be used to calculate the risk (in that sense) associated with any value of $P*$, and the value with the lowest risk could be considered optimal. Although such an approach is quite objective, it does not completely eliminate the subjective nature of setting $P*$ because it necessitates placing values on socioeconomic events such as fishery closures, changes in catch per effort, greater or lesser variability of annual catch, and recreational and esthetic factors. Nonetheless, such a procedure is quite different from setting $P*$ empirically, and given the economic data and assumptions needed, the approach could be useful.

Reference Points, Implementation, and Data Collection

The use of CM or REPAST is in essence the application of a control rule (in the sense of Restrepo et al. 1998) for the management of fisheries. Like other approaches in which added precision in the estimates of stock status yields a smaller margin between target and limit, REPAST makes evident the returns expected from expenditures on collecting relevant data (whether fishery dependent or not) because the quality of the data will determine the variance in the estimation of the LRP used. As noted by Caddy and McGarvey (1996), an important consequence of this is that with a higher level of expenditure on monitoring the same probability of exceeding the limit reference point occurs at a slightly higher rate of fishing than with the higher variability in LRP coming from less intensive monitoring. This makes explicit what was only implicit previously, namely, that statistical monitoring has an economic value to the fishing industry. Similarly, the value of enforcement and compliance becomes more apparent, as they serve to reduce implementation uncertainty (the variability in R_{next}), which also can reduce the required margin between target and limit.

The methods described here (including the CM method) assume explicitly that once adopted, a target will be met on average. Experience suggests, however, that quotas (for example) are much more likely to be exceeded than fallen short of. From the scientific perspective, there is no reason that the statistical distribution of F_{next} must be centered on the target, as we have assumed. Any of the methods described can easily accommodate distributions centered at any point. Therefore, any of the methods is easily extended to allow for expected overruns. In such applications, the central tendency and dispersion of F_{next} or R_{next} might be estimated from data on the performance of the fishery, as we suggested that the dispersion alone might be. In applying such estimates, it might be desirable to use a running average of (for example) the last few years' performance, so that changes in implementation effectiveness would be reflected in the new targets.

Application in Management

How can methods like REPAST best be used in ongoing fishery management? We suggest that they must be applied repeatedly because monitoring and assessment techniques change over time, as does the status of the stock itself. The limit reference point is most usefully set as a theoretical quantity (e.g., F_{MSY}) rather than as a specific value of F or B, as formally adopting a specific number often leads to difficulties when assessment methods change or even as knowledge about the stock increases. The acceptable probability $P*$ of overshooting the LRP can also be established before assessment takes place. For application of REPAST, stock assessment results should include estimates of the relative LRP and its CV. A complementary analysis of the fishery's past performance can be used to estimate implementation uncertainty in management measures (e.g., the uncertainty in F_{next}). From those estimates, REPAST is used to compute a relative TRP for the following period. The time of assessment would also be an excellent time for bioeconomic analyses, based on REPAST, of the costs and benefits of reducing uncertainty. The possible extra yield from reduced estimation uncertainty is balanced against the costs of better stock monitoring and assessment, which lower the variability in the LRP, and against those of better enforcement and fishery monitoring, which lower the variability in implementing the TRP. In summary, at each assessment cycle, REPAST allows the computation of targets from stock and fishery status and provides the opportunity to balance possible larger yields against the costs needed to attain them.

It is a property of all estimation schemes for fish and wildlife conservation, including estimates of reference points, that they are merely advisory. Remedial action, if needed, must be undertaken by other means: law, regulation, or other agreement. The value of reference points, it seems to us, is in establishing a framework within which such agreements can be made and, in the case of the methods proposed here, in offering a logical method for determining the magnitude of the adjustments that can be agreed upon. Thus, if accepted with some fidelity, such methods seem capable of furthering conservation management considerably.

Software

The authors have developed Fortran software implementing the REPAST scheme. This software, including the source code, will be made available free to colleagues requesting it. Contact M. H. Prager at mike.prager@noaa.gov.

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